A Necessary Set of Turns to Solve a Rubik's Cube

Kenny Roffo

Advisors: Bonita Graham and David Vampola

SUNY Oswego

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The Cube

Definition

A scramble is an arrangement of the pieces of the cube that is solvable by turning the faces of the cube

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- The Rubik's Cube has $43,252,003,274,489,856,000$ scrambles

The Six Turns

- The cube has 6 faces:

The Six Turns

- The cube has 6 faces:
    - **Right**

The Six Turns

- The cube has 6 faces:
  - **Right**
  - **Left**

The Six Turns

- The cube has 6 faces:
  - **Right**
  - **Left**
  - **Front**

The Six Turns

- The cube has 6 faces:
  - **Right**
  - **Left**
  - **Front**
  - **Back**

The Six Turns

- The cube has 6 faces:
  - **Right**
  - **Left**
  - **Front**
  - **Back**
  - **Up**

The Six Turns

- The cube has 6 faces:
  - **Right**
  - **Left**
  - **Front**
  - **Back**
  - **Up**
  - **Down**

The Six Turns

- The cube has 6 faces:
  - **Right**
  - **Left**
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  - **Back**
  - **Up**
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The Six Turns

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  - **Right**
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The Six Turns

- The cube has 6 faces:
  - **Right**
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  - **Up**
  -

The Six Turns

- The cube has 6 faces:
  - **Right** - R
  - **Left** - L
  - **Front** - F
  - **Back** - B
  - **Up** - U
  - - $\mathbb{D}$

The Six Turns

- The cube has 6 faces:
  - **Right** - R
  - **Left** - L
  - **Front** - F
  - **Back** - B
  - **Up** - U
  - - $\mathbb{D}$
- 90 degree clockwise rotation

The Six Turns

- The cube has 6 faces:
  - **Right** - R
  - **Left** - L
  - **Front** - F
  - **Back** - B
  - **Up** - U
  - - $\mathbb{D}$
- 90 degree clockwise rotation
- R, L, F, B, U, $\mathbb{D}$ are permutations

Examples

$$R\ U^2\ \mathbb{D}^2\ U^{-1}\ \mathbb{D}^2\ U^{-1}\ R^{-1}$$

Examples

- The Identity Permutation:

$$R\ U^2\ \mathbb{D}^2\ U^{-1}\ \mathbb{D}^2\ U^{-1}\ R^{-1}$$

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- Consider an “A-Perm”:

$$R^{-1}\ F\ R^{-1}\ B^2\ R\ F^{-1}\ R^{-1}\ B^2\ R^2$$

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- The Identity Permutation:

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- Consider an “A-Perm”:

$$R^{-1}\ F\ R^{-1}\ B^2\ R\ F^{-1}\ R^{-1}\ B^2\ R^2$$

- The inverse of this permutation is:

$$R^2\ B^2\ R\ F\ R^{-1}\ B^2\ R\ F^{-1}\ R$$

The Conditions for Generators

- The face turns can be considered the generators of the Rubik's Cube group

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Proposition

The generators R, L, F, B and U will be necessary and sufficient to solve any scramble of a Rubik's Cube.

Necessity of 5 generators

Consider cases:

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- Restrict two adjacent faces

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Consider cases:

- Restrict two adjacent faces
- Restrict two opposite faces

Necessity: Restricting Two Adjacent Faces

- Without loss of generality, we will consider the U and F face turns to be restricted

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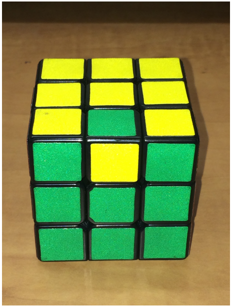

Necessity: Restricting Two Opposite Faces

- Without loss of generality, we will consider the U and $\mathbb{D}$ face turns to be restricted

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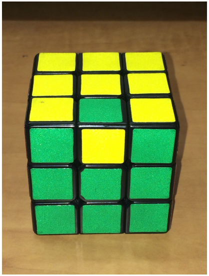

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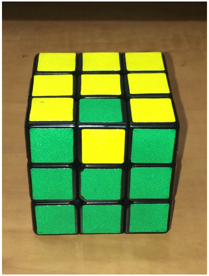

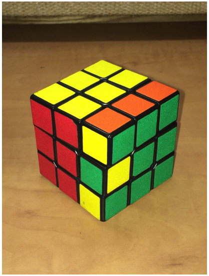

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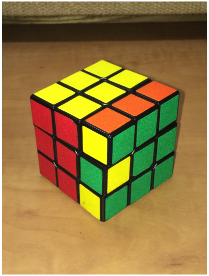

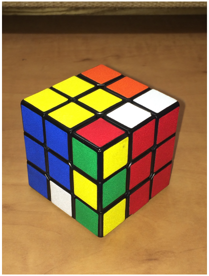

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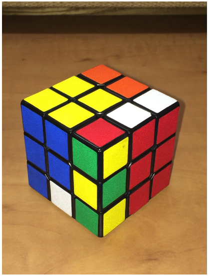

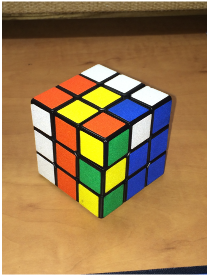

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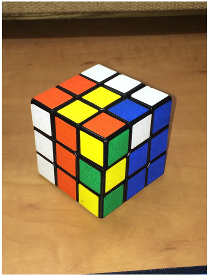

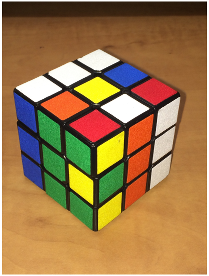

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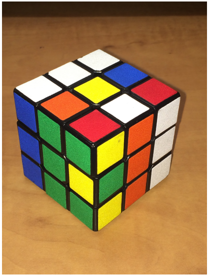

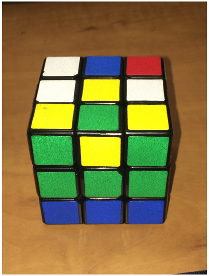

The Big Permutation

$$(R^2\ L^2\ U^{-1}\ F^2\ B^2)\ U^{-1}\ R^{-1}\ L\ U^2\ R^2\ L^2\ U^2\ R^2\ U^2\ R^2\ L^2\ U^2\ L$$
$$(R^{-1}\ U^2\ R^2\ U^2\ R^2\ U^2)\ (L^2\ U^2\ L^2\ U^2\ L^2\ U^2)$$
$$(F^2\ U^2\ F^2\ U^2\ F^2\ U^2)\ (B^2\ U^2\ B^2\ U^2\ B^2\ U^2)$$
$$(R\ U\ R^{-1}\ U^{-1}\ R^{-1}\ F\ R^2\ U^{-1}\ R^{-1}\ U^{-1}\ R\ U\ R^{-1}\ F^{-1})\ U$$
$$(R^{-1}\ F\ R^{-1}\ B^2\ R\ F^{-1}\ R^{-1}\ B^2\ R^2)\ U$$

Video of the D Permutation

Youtube Link to Video

The Big Permutation

$$(R^2\ L^2\ U^{-1}\ F^2\ B^2)\ U^{-1}\ R^{-1}\ L\ U^2\ R^2\ L^2\ U^2\ R^2\ U^2\ R^2\ L^2\ U^2\ L$$
$$(R^{-1}\ U^2\ R^2\ U^2\ R^2\ U^2)\ (L^2\ U^2\ L^2\ U^2\ L^2\ U^2)$$
$$(F^2\ U^2\ F^2\ U^2\ F^2\ U^2)\ (B^2\ U^2\ B^2\ U^2\ B^2\ U^2)$$
$$(R\ U\ R^{-1}\ U^{-1}\ R^{-1}\ F\ R^2\ U^{-1}\ R^{-1}\ U^{-1}\ R\ U\ R^{-1}\ F^{-1})\ U$$
$$(R^{-1}\ F\ R^{-1}\ B^2\ R\ F^{-1}\ R^{-1}\ B^2\ R^2)\ U$$

The Big Permutation

$$(R^2\ L^2\ U^{-1}\ F^2\ B^2)\ U^{-1}\ R^{-1}\ L\ U^2\ R^2\ L^2\ U^2\ R^2\ U^2\ R^2\ L^2\ U^2\ L$$
$$(R^{-1}\ U^2\ R^2\ U^2\ R^2\ U^2)\ (L^2\ U^2\ L^2\ U^2\ L^2\ U^2)$$
$$(F^2\ U^2\ F^2\ U^2\ F^2\ U^2)\ (B^2\ U^2\ B^2\ U^2\ B^2\ U^2)$$
$$(R\ U\ R^{-1}\ U^{-1}\ R^{-1}\ F\ R^2\ U^{-1}\ R^{-1}\ U^{-1}\ R\ U\ R^{-1}\ F^{-1})\ U$$
$$(R^{-1}\ F\ R^{-1}\ B^2\ R\ F^{-1}\ R^{-1}\ B^2\ R^2)\ U$$

$$= \mathbb{D}$$